Systems

Lecture #3

1.3

Homework

- Linear Systems
  - Is $y(t)=x(t)^2$ a linear system? Prove your point.
  - Is $y(t)=t^2$ a linear system? Prove your point.
  - CT.1.3.1
- ODE
  - Solve and plot the solution to the equation: $dx/dt + 6\,x = 0$; $x(0) = 5$; use Matlab to obtain the plot
  - Solve and plot the solution to the equation : $dx/dt + 6\,x = 6$; $x(0) = 0$; use Matlab to obtain the plot

Homework

- BioSignals
  - An heart signal is sampled at the rate 250 s/s and is passed to EKG which has an input consisting of a low pass filter. The filter is a resistor and capacitor in series where the output of the filter is taken across the capacitor. What should be the value of the Capacitor if the Resistor is 1k ohms and the time constant of the filter so that the transient response is completed within 1/10 of the sample time?

Homework Answers

- Linear Systems
  - Is $y(t)=x(t)^2$ a linear system? Prove your point.
  - Is $y(t)=t^2$ a linear system? Prove your point.

$$y_a(t) = x_a(t)^2; y_b(t) = x_b(t)^2$$

$$y_{sum}(t) = [x_a(t) + x_b(t)]^2 = x_a(t)^2 + 2x_a(t)x_b(t) + x_b(t)^2 \neq y_a(t) + y_b(t)$$

NOT LINEAR

$$y(t_a) = t_a{}^2; y(t_b) = t_b{}^2;$$

$$y(t_a + t_b) = [t_a + t_b]^2 = t_a{}^2 + 2t_a t_b + t_b{}^2 \neq y(t_a) + y(t_b)$$

NOT LINEAR

CT1.3.1

$y(t) = A_c[1 + mx(t)]\cos(2\pi F_c t + \phi)$

Linear

$x_1 \Rightarrow A_c[1 + mx_1(t)]\cos(2\pi F_c t + \phi) = y_1(t)$

$a_1 x_1 \Rightarrow A_c[1 + ma_1 x_1(t)]\cos(2\pi F_c t + \phi)$

$a_1 y_1(t) = a_1 A_c[1 + mx_1(t)]\cos(2\pi F_c t + \phi) = A_c[a_1 + ma_1 x_1(t)]\cos(2\pi F_c t + \phi) \neq A_c[1 + ma_1 x_1(t)]\cos(2\pi F_c t + \phi)$

NOT Linear

Time Invariant

$x_1(t) \Rightarrow A_c[1 + mx_1(t)]\cos(2\pi F_c t + \phi) = y_1(t)$

$x_1(t - \tau) \Rightarrow A_c[1 + mx_1(t - \tau)]\cos(2\pi F_c t + \phi)$

$y_1(t - \tau) = A_c[1 + mx_1(t - \tau)]\cos(2\pi F_c\{t - \tau\} + \phi) \neq A_c[1 + mx_1(t - \tau)]\cos(2\pi F_c t + \phi)$

NOT Time Invariant

Causal

$y(t_o) = A_c[1 + mx(t_o)]\cos(2\pi F_c t_o + \phi)$ only depends on values of $x(t_o)$

YES Causal

Stable

$y(t) = A_c[1 + mx(t)]\cos(2\pi F_c t + \phi)$ since $x(t)$ is bounded then $y(t)$ is bounded since $\cos(2\pi F_c t + \phi)$ and A_c are also bounded

YES Stable

Homework Answers #2

ODE

- Solve the equation: dx/dt + 6 x = 0
- Solve the equation: dx/dt + 6 x = 6

$$\frac{dx}{dt} + 6x = 0$$

$$\text{Choose}: \ x(t) = Ke^{\alpha t}$$

$$K\alpha e^{\alpha t} + 6Ke^{\alpha t} = 0$$

$$\alpha = -6$$

$$x(t) = Ke^{-6t}$$

$$K = x(0)$$

$$\frac{dx}{dt} + 6x = 6$$

$$\text{Choose}: \ x(t) = K_1 e^{\alpha t} + K_2$$

$$K_1 \alpha e^{\alpha t} + 6(K_1 e^{\alpha t} + K_2) = 6$$

$$K_1 \alpha e^{\alpha t} + 6K_1 e^{\alpha t} = 0; \ 6K_2 = 6$$

$$\alpha = -6; K_2 = 1$$

$$x(t) = K_1 e^{-6t} + 1$$

$$K_1 = x(0) - 1$$

Matlab Code

```
time=(0:.1:5);
K=5;
x=K*exp(-6*time);
subplot(2,1,1);
plot(time,x);
title('Continuous Plot');
xlabel('Seconds');
axis([time(1) time(length(time)) min(x)*1.1 max(x)*1.1]);
x=1-exp(-6*time);
subplot(2,1,2);
plot(time,x);
title('Continuous Plot');
xlabel('Seconds');
axis([time(1) time(length(time)) min(x)*1.1 max(x)*1.1]);
```